Menofiya University		**Year: Third** **Code : MPE322**
Faculty of Engineering		**Department: MPE**
Shebin El-Kom		**Subject: Numerical methods in MPE**
Second semester final exam		**Time Allowed: 3 hours**
Academic Year: 2012-2013	Minufiya University	**Date: 15 / 6 / 2013**

Question (1) **(25 Marks)**

(a) By using the recurrence formula, Write a computer code to compute $\partial^3 f/\partial x^3$ with central finite difference technique at x=2 when $f = 2\ln x$. Take $\Delta x = 0.1$. Compare the result with analytical differentiation.

(8 marks)

(b) Prove that, there is a root lies between x=1 and x=2 for the following

$x^3 - 0.5 = 0.8x$

Find numerical the root by using false position method, then write a computer code to obtain this root.

(8 Marks)

(c) By using Polynomial method, derive finite difference formula to represent 2nd derivative, 1st order accuracy with backward FD technique. Consider expanding grid

(9 marks)

Question (2) **(20 Marks)**

(a) Starting with Taylor series, derive the 2nd derivative central finite difference approximation with second order accuracy. Calculate the numerical and analytical values of this derivative for the following function and assign the relative numerical error %

$f = 5e^{0.2x}$ .take $\Delta x = 0.01$

(8marks)

(b) The dissociation of chemical species A_2 is as follows

$$A_2 \overset{K_f}{\Rightarrow} 2A$$

The rate of dissociation is represented by the following equation.

$$\frac{dC_{A_2}}{dt} = -k_f C_{A_2}$$

Where C_{A_2} is the instantaneous concentration of species A_2 in mol/l and k_f is the forward reaction rate constant = $1 \times 10^3\ s^{-1}$.

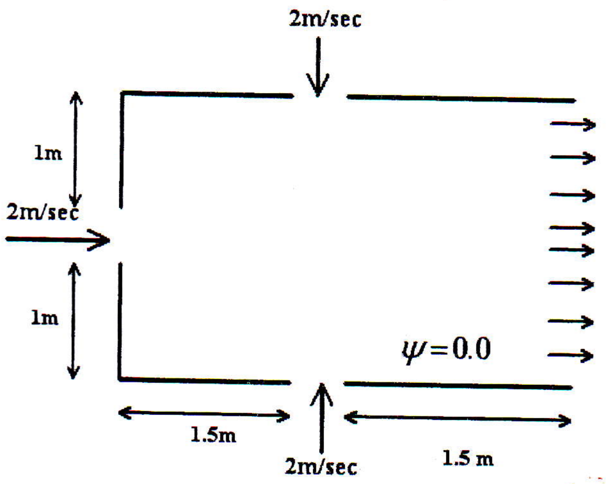
If $A_2(0) = 1$ write a program to calculate the time required to dissociate A_2 to its half initial concentration.

Take step size equal to 1×10^{-5} s. Compare the results with analytical solution.

(12marks)

Question (3) **(20 Marks)**

The ideal incompressible flow inside a rectangular chamber can be described by $\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}=0$, where ψ is the stream function. The flow enters the chamber from three ports each 0.3m wide at a velocity of 2m/s, as shown in the figure. The velocity distribution through the nozzle can be calculated from $u=\frac{\partial\psi}{\partial y}$ and $v=-\frac{\partial\psi}{\partial x}$, where u and v are the horizontal and vertical velocity component, respectively.



a. Evaluate the boundary conditions of this problem.
b. Describe the solution procedure of this equation using the line-Seidel method
c. Write a computer program for the solution procedure described in b.
d. If the pressure at the left port is 1 bar, calculate the pressure distribution within the chamber.

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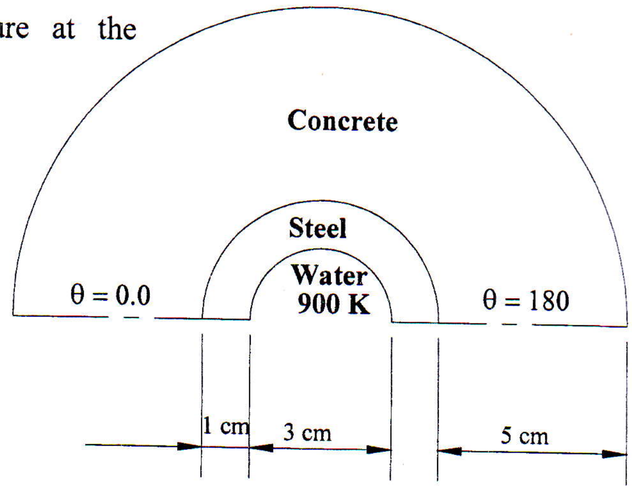
Question (4) **(20 Marks)**

In pressurized-water nuclear reactor water flows in two-layer tube of inner diameter 3cm at constant temperature of 900 K. The inner layer is made of steel having thermal diffusivity of 0.9 m^2/s and has a thickness of 1 cm. The outer layer is made of concrete of thermal diffusivity of 0.3 m^2/s and has a thickness of 5 cm, see the figure. The differential equation governs the heat transfer through the wall can be written as: $\frac{\partial T}{\partial t}=\frac{1}{r}\frac{\partial}{\partial r}\left(r\alpha\frac{\partial T}{\partial r}\right)$

where, k is the local thermal conductivity of the wall. The temperature at the outer surface can be taken as 300 K. At the interface $\left(-k\,\partial T/\partial r\right)_{steel}=\left(k\,\partial T/\partial r\right)_{concrete}$. Where, k is the thermal conductivity. Use $\Delta r = 0.2$ mm for steel and $\Delta r = 0.5$ mm for concrete, Answer the following

a. Describe the solution procedure using Rechardson method
b. Show with some detailed how the temperature at the interface is calculated.
c. Draw a flowchart for the solution procedure.
d. Comment on the stability of this method.



GOOD LUCK

Assoc. Prof. Safwat A. Wilson **Dr. Samy M. El-Behery**